CP violation and chiral symmetry restoration in the hot linear sigma model in a strong magnetic background

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We study the effects of CP violation on the nature of the chiral transition within the linear sigma model with two flavors of quarks. The finite-temperature effective potential containing contributions from nontrivial values for the parameter θ is computed to one loop order and their minima structure is analyzed. Motivated by the possibility of observing the formation of CP-odd domains in high-energy heavy ion collisions, we also investigate the behavior of the effective potential in the presence of a strong magnetic background. We find that the nature of the chiral transition is influenced by both θ and the magnetic field.

I. INTRODUCTION

In Quantum Chromodynamics (QCD), the existence of instanton configurations for the gauge fields [1] and their intimate connection with the axial anomaly [2] allows for a nontrivial topological term in the action. This term is usually neglected, since it breaks the original CP symmetry and so far experiments indicate that its coefficient, known as the θ parameter, is vanishingly small, $\theta \lesssim 10^{-10}$ [3] (see also [4]). The reason why θ is so small (or zero) is unclear, and this issue is called the *strong CP problem* [1].

Although it has been proved that CP can not be spontaneously broken in the vacuum of QCD for $\theta = 0$ [6], this theorem might not hold for QCD matter at finite temperature or density [7]. With this premise, it has been proposed in Ref. [8] that hot matter produced in ultra-relativistic heavy ion collisions could exhibit domains of metastable states that violate CP. These states could be described by a QCD action that incorporates the topological θ-term, and would decay via CP-odd processes. Possible experimental signatures for the presence of CP-odd domains are based on charge separation of hadronic matter produced in heavy ion collisions [9]. This effect is enhanced by the presence of a strong magnetic background in the case of noncentral collisions, as was realized more recently, by a mechanism that has been called *chiral magnetic effect* [10], and could in principle be observed a RHIC and the LHC.

In this paper we investigate how the chiral transition is affected when the ingredients mentioned above, i.e. CP-odd effects and a strong magnetic background, are present, as in the case of noncentral heavy-ion collisions. For this purpose, we adopt the linear sigma model coupled with two flavors of quarks as our effective theory [11] to study the chiral transition [2]. To include the effects of the presence of the axial anomaly and CP violation, we add a term that mimics the presence of nontrivial gauge field configurations, the 't Hooft determinant [26]. The latter is a function of the parameter θ, and is responsible for CP violation for non-vanishing values of θ. In this framework, the presence of CP violation is directly related to a nonzero η condensate.

The rich vacuum structure brought about by a nonzero θ term in the action [27] has an influence on the chiral transition, and generates a more complex picture in the analysis of the phase diagram of strong interactions [3]. As will be shown below, three phases emerge: one with σ and η condensates, another where the η condensate vanishes and the σ condensate remains, and a phase where both condensates are melted. The topography of extrema is therefore rich, and metastable minima do appear in certain situations.

Once we have an effective theory describing chiral symmetry restoration at finite temperature in the presence of CP violation, we study how this CP-odd linear sigma model is affected by the presence of a strong magnetic background, as is presumably generated in the case of noncentral high-energy heavy ion collisions [10]. This extends the analysis we presented in Ref. [29], where we investigated the effects of a strong magnetic background on the nature and dynamics of the chiral phase transition at finite temperature and vanishing chemical potential, and found that the nature of the chiral transition is modified. Here we find that the main result obtained in [29], namely that the presence of a strong magnetic background turns the crossover into a weak first-order phase

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[1] Frameworks that preserve the CP symmetry were also proposed, the most studied being axion theories [5], but those were not confirmed by experiments to date either, leaving the problem unsolved.

[2] This effective theory, especially the σ-π-quark sector, has been widely used to describe different aspects of the chiral transition, such as thermodynamic properties [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] and the nonequilibrium phase conversion process [23], as well as combined to other models in order to include effects from confinement [24, 25], usually without the inclusion of the θ term.

[3] See, for instance, Ref. [28] for a detailed study of the phase structure of the two-flavor NJL model at $\theta = \pi$.

transition, is maintained. In fact, the combination of effects from high magnetic fields and the CP-odd contribution allows for nontrivial combinations of metastable CP-odd and chirally symmetric phases, a structure that might be relevant for the supercooling dynamics that presumably happens after a high-energy heavy ion collision.

Modifications in the vacuum of CP-symmetric QCD by the presence of a magnetic field have been investigated previously within different frameworks, mainly using effective models [30, 31, 32, 33, 34, 35, 36, 37], especially the NJL model [38], and chiral perturbation theory [39, 40, 41], but also resorting to the quark model [42] and certain limits of QCD [43]. Most treatments have been concerned with vacuum modifications by the magnetic field, though medium effects were considered in a few cases, as e.g. in the study of the stability of quark droplets under the influence of a magnetic field at finite density and zero temperature, with nontrivial effects on the order of the chiral transition [44]. More recently, magnetic effects on the dynamical quark mass [45] and on the thermal quark-hadron transition [46], as well as magnetized chiral condensates in a holographic description of chiral symmetry breaking [47], were also considered.

The paper is organized as follows. Section II presents the low-energy effective model adopted in this paper to investigate the chiral transition in the presence of CP violating terms. In Section III we show our results for the effective potential at finite temperature, and discuss the condensates. In section IV we incorporate effects coming from the presence of a strong magnetic background. Section V contains our conclusions and outlook.

II. CP-ODD LINEAR SIGMA MODEL

The classical Lagrangian for QCD is invariant under parity, P, and charge conjugation, C, transformations. However, at a quantum level these symmetries are not preserved. This breaking manifests itself as the axial (or ABJ) anomaly [2], and the current associated with this symmetry is no longer conserved, even in the massless limit for quarks:

$$\partial_\mu J_5^\mu = 2m_f i\bar{\psi}_f\gamma_5\psi_f - \frac{N_f g^2}{16\pi^2}F^{\mu\nu a}\tilde{F}^a_{\mu\nu} , \tag{1}$$

where g is the gauge coupling, N_f is the number of quark flavors with masses m_f, $\tilde{F}$ is the dual of the gauge field strength tensor F, and sums over color indices a and flavor indices f are implicit. Naively, the last term in the expression above would be irrelevant, since it can be written as a total derivative. However, if one considers topologically nontrivial gauge field configurations, this term is nonvanishing and can be associated with the winding number of each configuration. Each sector of configurations, with a given winding number, has its own vacuum, and the true vacuum of the theory becomes a superposition of the several inequivalent topological vacua $|\theta\rangle = \sum_n e^{-in\theta}|n\rangle$, where θ is a free parameter. In this context, to calculate expectation values one has to compute an average over a sector with fixed winding number and then sum over all sectors. This procedure is equivalent to adding a topological term in the Lagrangian, as follows:

$$\mathcal{L}_{QCD} = \mathcal{L}_{cl} - \frac{\theta}{32\pi^2}g^2F^{\mu\nu a}\tilde{F}^a_{\mu\nu} . \tag{2}$$

To describe the chiral phase structure of strong interactions including CP-odd effects, we adopt an effective model that reproduces the symmetries of QCD at low energy scales, and has the appropriate degrees of freedom at each scale: the CP-odd linear sigma model coupled with two flavors of quarks. The chiral mesonic sector is built including all Lorentz invariant terms allowed by symmetry and renormalizability. Following Refs. [26, 48], one can write

$$\begin{aligned}\mathcal{L}_\chi &= \frac{1}{2}\mathrm{Tr}(\partial_\mu\phi^\dagger\partial^\mu\phi) + \frac{a}{2}\mathrm{Tr}(\phi^\dagger\phi) - \frac{\lambda_1}{4}[\mathrm{Tr}(\phi^\dagger\phi)]^2 \\ &\quad -\frac{\lambda_2}{4}\mathrm{Tr}[(\phi^\dagger\phi)^2] + \frac{c}{2}[e^{i\theta}\det(\phi) + e^{-i\theta}\det(\phi^\dagger)] \\ &\quad +\mathrm{Tr}[h(\phi+\phi^\dagger)] .\end{aligned} \tag{3}$$

The potential in the Lagrangian above displays both spontaneous and explicit symmetry breaking, the latter being implemented by the term $\sim h$. The strength of CP violation is contained in the 't Hooft determinant term, which encodes the Levi-Civita structure of the axial anomaly and depends on the value of the parameter θ.

Expressing the chiral field ϕ as

$$\phi = \frac{1}{\sqrt{2}}(\sigma + i\eta) + \frac{1}{\sqrt{2}}(\vec{\alpha} + i\vec{\pi})\cdot\vec{\tau} , \tag{4}$$

the potential takes the following form (substituting the parameter h by $H \equiv \sqrt{2}h$):

$$\begin{aligned}V_\chi &= -\frac{a}{2}(\sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{\alpha}_0^2) \\ &\quad -\frac{c}{2}\cos\theta\,(\sigma^2 + \vec{\pi}^2 + \eta^2 + \vec{\alpha}_0^2) \\ &\quad +c\,\sin\theta\,(\sigma\eta - \vec{\pi}\cdot\vec{\alpha}_0) - H\sigma \\ &\quad +\frac{1}{4}(\lambda_1 + \frac{\lambda_2}{2})(\sigma^2 + \eta^2 + \vec{\pi}^2 + \vec{\alpha}_0^2)^2 \\ &\quad +\frac{2\lambda_2}{4}(\sigma\vec{\alpha}_0 + \eta\vec{\pi} + \vec{\pi}\times\vec{\alpha}_0)^2 .\end{aligned} \tag{5}$$

The parameters a, c, H, λ_1 and λ_2 can be fixed by vacuum properties of the mesons, so that the effective model reproduces correctly the phenomenology of QCD at low energies and in the vacuum, for vanishing θ, such as the spontaneous (and small explicit) breaking of chiral symmetry and experimentally measured meson masses.

In our treatment, the chiral mesons are coupled in the standard Yukawa fashion to two flavors of quarks. The latter constitute a thermalized fluid that provides a background in which the long wavelength modes of the chiral

fields evolve. The full Lagrangian has the form:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma)^2+\frac{1}{2}(\partial_\mu\vec{\pi})^2+\frac{1}{2}(\partial_\mu\eta)^2+\frac{1}{2}(\partial_\mu\vec{\alpha}_0)^2\\ &- V(\sigma,\eta,\vec{\pi},\vec{\alpha}_0)+\bar{\psi}_f(i\gamma^\mu\partial_\mu)\psi_f\\ &- g\bar{\psi}_f(-i\gamma^5\vec{\tau}\cdot\vec{\pi}+\sigma\gamma^5\vec{\tau}\cdot\vec{\alpha}_0-i\gamma^5\eta)\psi_f\ . \end{aligned} \tag{6}$$

For $T=0$ quarks degrees of freedom are absent (excited only for $T>0$), and results from chiral perturbation theory for the broken phase vacuum are reproduced [49]. For $T>0$, quarks are relevant (fast) degrees of freedom and chiral symmetry is approximately restored in the plasma for high enough T. In this case, we incorporate quark thermal fluctuations in the effective potential for the mesonic sector, i.e. we integrate over quarks to one loop.

To one loop, and within a classical approximation for the chiral field background, a standard functional integration over the fermions [49] gives an effective potential of the form $V_{eff}=V(\phi)+V_q(\phi)$, where the quark contribution is given by

$$V_q \equiv -\nu_q T\int\frac{d^3k}{(2\pi)^3}\ln\left(1+e^{-E_k(\phi)/T}\right)\ . \tag{7}$$

where $\nu_q=24$ is the color-spin-isospin-baryon charge degeneracy factor, and $E_k(\phi)=(\vec{k}^2+M_q^2(\phi))^{1/2}$, M_q playing the role of an effective mass for the quarks.

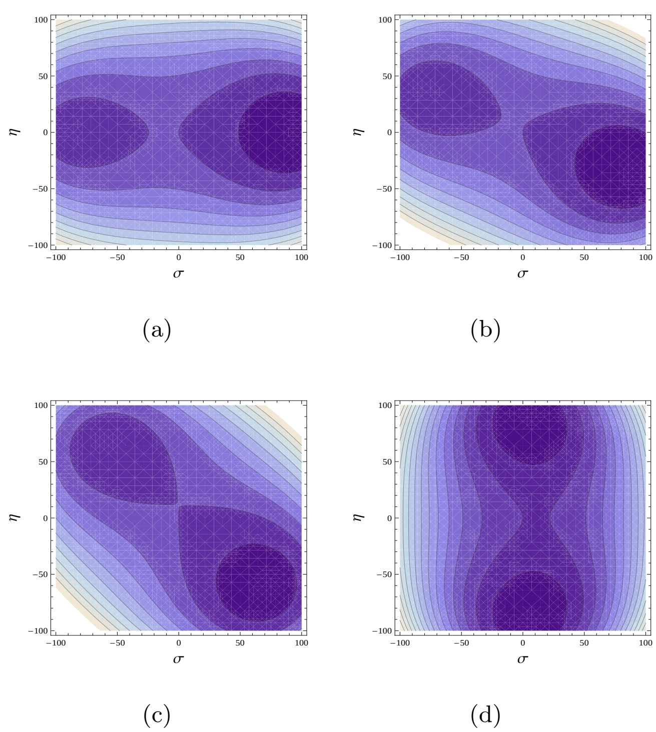


FIG. 1: Contour plots of the effective potential in the vacuum. (a) $\theta=0$; (b) $\theta=\pi/4$; (c) $\theta=\pi/2$; (d) $\theta=\pi$. The numerical values are in MeV.

Following a mean field analysis, we take $\sigma=\langle\sigma\rangle+\sigma'$ and $\eta=\langle\eta\rangle+\eta'$, and assume that the remaining condensates vanish. In this approximation, the effective mass for the quarks is simply given by $M_q=g\sqrt{\langle\sigma\rangle^2+\langle\eta\rangle^2}$. The effective potential can then be split into a classical piece and its fluctuation corrections, besides the contribution coming from the quarks, i.e., $V_{eff}=V_\chi^{cl}+V_\chi^{fluct}+V_q$. The classical contribution has the form

$$\begin{aligned}V_\chi^{cl} &= \frac{\lambda}{4}(\langle\sigma\rangle^2-v_\theta^2)^2-H\langle\sigma\rangle\\ &+ \frac{\lambda}{4}(\langle\eta\rangle^2-u_\theta^2)^2-c\ \sin\theta\ \langle\sigma\rangle\langle\eta\rangle\\ &+ \frac{\lambda}{2}\langle\sigma\rangle^2\langle\eta\rangle^2-\frac{\lambda}{4}(v_\theta^4+u_\theta^4)\end{aligned} \tag{8}$$

where we have defined the combination of couplings $\lambda\equiv\lambda_1+\lambda_2/2$, and the following auxiliary quantities:

$$v_\theta^2\equiv\frac{a+c\ \cos\theta}{\lambda}\quad ;\quad u_\theta^2\equiv v_\theta^2-\frac{2c}{\lambda}\cos\theta\ . \tag{9}$$

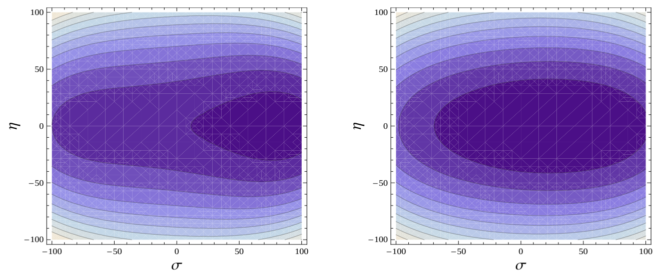
The fluctuation correction, up to quadratic terms, is straightforwardly worked out and provides the means to fix the parameters of the model to meson masses and the pion decay constant f_π in the vacuum in the absence of CP violation. Concretely, we impose that in the vacuum chiral symmetry is spontaneously broken and the expectation value of the chiral condensate is given by $\langle\sigma\rangle=f_\pi=93$ MeV. The coefficient of the term that breaks explicitly chiral symmetry is given by the PCAC relation $H=f_\pi m_\pi^2$. All the other parameters are fixed to reproduce the mesons masses: $m_\sigma=600$ MeV, $m_\eta=574$ MeV, $m_\pi=138$ MeV, and $m_{\alpha_0}=980$ MeV.

This completes the setup of the CP-odd linear sigma model to be used as an effective theory in the investigation of the chiral transition in the presence of nontrivial CP violating processes. In what follows we present our results for the effective potential for representative values of the parameter θ and the temperature. Effects from a strong magnetic background will be incorporated later.

III. RESULTS FOR THE EFFECTIVE POTENTIAL

The effective potential we obtained in the previous section is a function of the condensates of σ and η, and of the CP violation parameter θ. Below we analyze the dependence of the effective potential on θ for different values of the temperature. In the vacuum, i.e. in the zero temperature case, the potential has two minima in the σ direction for $\theta=0$, as expected from usual (CP-even) linear sigma model. The contour plot for the effective potential in the plane $\langle\sigma\rangle-\langle\eta\rangle$ for this situation is presented in Fig. 1(a). As we increase θ, the pair of minima rotates, and for $\theta=\pi$ it is almost in the $\langle\eta\rangle$ direction, as can be seen in Figs. 1(b) and (c). Although this is not a realistic (physical) case, since nonzero values of θ are not observed in the vacuum of strong interactions, it

is nevertheless interesting to study all the phase space to have a complete map of the dependence on θ.



(a) (b)

FIG. 2: Contour plots of the effective potential for $\theta = 0$. (a) $T = 120$ MeV; (b) $T = 160$ MeV. The numerical values are in MeV.

Keeping $\theta = 0$ and increasing the temperature, the two minima remain in the $\langle\sigma\rangle$ direction, as expected, and both minima approach to the origin, $\langle\sigma\rangle \approx 0$, so that for high enough temperatures chiral symmetry is approximately restored (see Fig. 2). Chiral symmetry is not completely restored due to the explicit breaking caused by the mass term ($\sim H$) in the effective potential. In Fig. 3, we plot the potential in the $\langle\sigma\rangle$ direction for $\theta = 0$ and different values of the temperature. As expected from the usual linear sigma model, the transition to the phase with approximately restored chiral symmetry is a crossover.

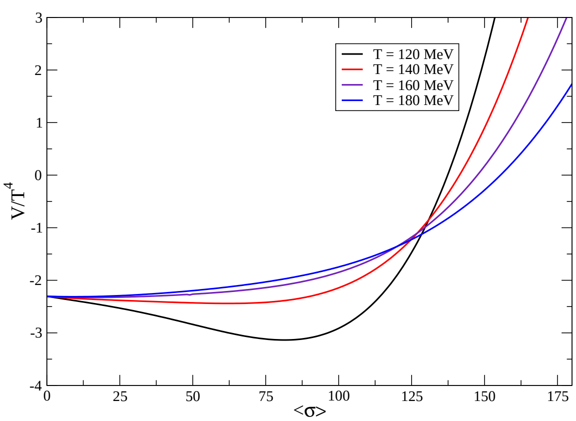
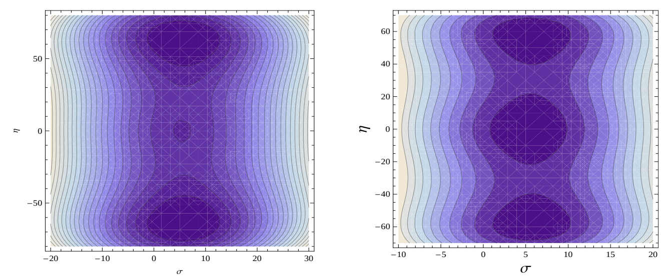


FIG. 3: Effective potential normalized by the temperature in the $\langle\sigma\rangle$ (in MeV) direction at $\theta = 0$ for several values of the temperature.

For $\theta = \pi$, the degenerate minima are almost in the $\langle\eta\rangle$ direction, as it is shown in Fig. 1(d). Increasing the temperature, the minima move towards the center, indicating a chiral symmetry restoration, as displayed in Fig. 4. However, in this case, there is a barrier between the global minimum and the new minimum that will become the true global minimum at high temperature at $\eta = 0$. In contrast to the previous case, this signals a first-order transition, and the possibility of metastable CP-odd states.



(a) (b)

FIG. 4: Contour plots of the effective potential for $\theta = \pi$. (a) $T = 125$ MeV; (b) $T = 128$ MeV. The numerical values are in MeV.

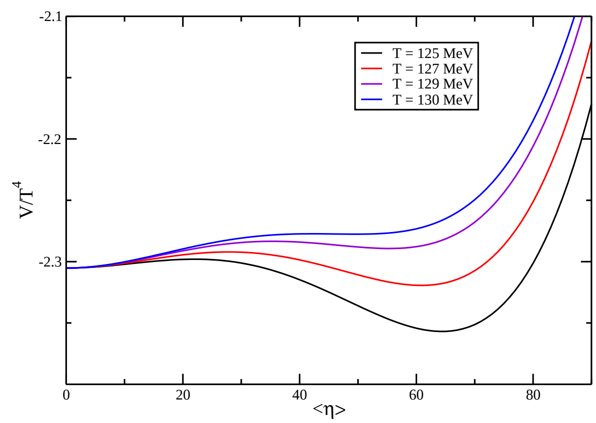


FIG. 5: Effective potential normalized by the temperature in the $\langle\eta\rangle$ (in MeV) direction at $\theta = \pi$ for different values of the temperature.

To show more clearly the barrier, we display in Fig. 5 the effective potential at $\theta = \pi$ for different values of the temperature. One can notice the new minimum emerging at $\langle\eta\rangle = 0$ for a temperature around $T = 126$ MeV. In this case, the critical temperature is lower than the critical temperature for the melting of the condensate $\langle\sigma\rangle$. Since the critical temperatures for the melting of $\langle\sigma\rangle$ and $\langle\eta\rangle$ are different, three different phases are allowed in systems with θ between zero and π: one in which

both condensates are present, another one where the $\langle\eta\rangle$ condensate vanishes, and a phase where both condensates vanish.

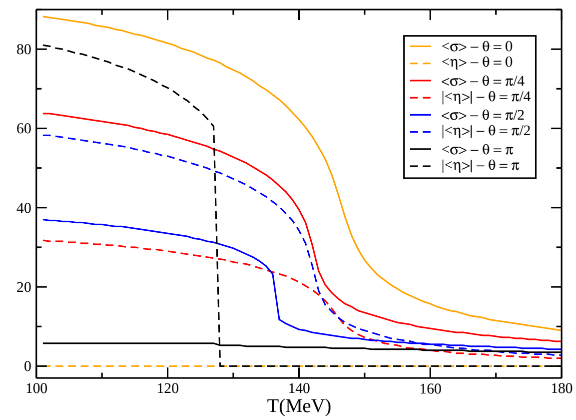
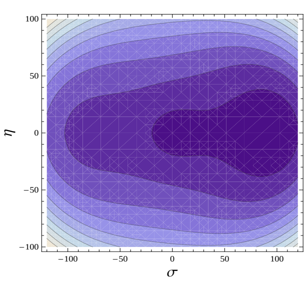
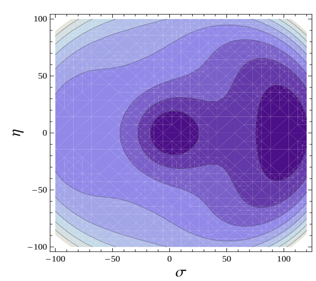


FIG. 6: Absolute value of the condensates (in MeV) as functions of the temperature. As θ approaches π, the transition in the $\langle\eta\rangle$ direction becomes stronger. Here full lines denote $\langle\sigma\rangle$ and dotted lines $\langle\eta\rangle$.



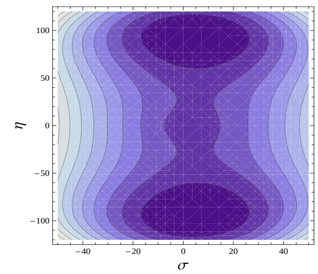
(a)



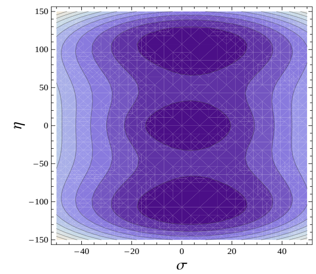
(b)

FIG. 7: Contour plots of the effective potential for $\theta = 0$ and different values of B. The temperature of each plot was chosen to be close to the respective critical temperature. (a) $B = 10m_\pi^2$; (b) $B = 20m_\pi^2$. The numerical values are in MeV.

In Fig. 6, we plot the absolute value of the expectation values of σ and η as functions of the temperature for different values of θ. Here, full lines stand for the σ expectation value and dashed lines represent the condensate $\langle\eta\rangle$. Since their values are "complementary", the curves for different values of θ can be paired using the value of each condensate at $T = 100$ MeV: for $\theta = 0$ the highest solid line corresponds to $\langle\sigma\rangle$ and the lowest dashed line to $\langle\eta\rangle$, for $\theta = \pi/4$ the second to highest solid line corresponds to $\langle\sigma\rangle$ and the second to lowest dashed line to $\langle\eta\rangle$, and so on. One can see that while $\langle\sigma\rangle$ decays smoothly for different values of θ, the condensate $\langle\eta\rangle$ decays sharply when θ approaches π, indicating first-order



(a)



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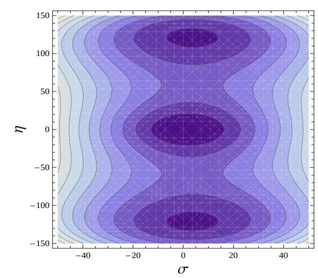
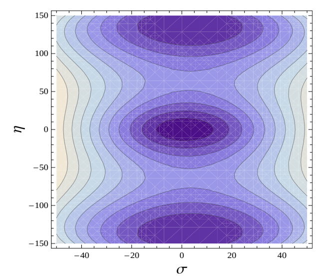


(c)



(d)

FIG. 8: Contour plots of the effective potential for $\theta = \pi$ and different values of B. The temperature was chosen to be $T = 195$ MeV. (a) $B = 5m_\pi^2$; (b) $B = 10m_\pi^2$; (c) $B = 15m_\pi^2$; (d) $B = 20m_\pi^2$. The numerical values are in MeV.

phase transition. It is also clear in the figure that the critical temperatures for the melting of the two condensates are different, allowing for the possibility of three different phases, as mentioned above.

IV. EFFECTS FROM A STRONG MAGNETIC BACKGROUND

Motivated by the possibility of observing the formation of CP-odd domains in high-energy heavy ion collisions in the presence of strong magnetic fields for noncentral collisions, we can incorporate the effect of a strong magnetic background on the effective potential for the CP-odd linear sigma model, and study how the chiral transition and the condensates are modified. Assuming that the system is now in the presence of a strong magnetic field background that is constant and homogeneous, one can compute the modified effective potential following the procedure we presented in detail in Ref. [29]. In what follows, we simply sketch the method and move to the discussion of the results.

For definiteness, let us take the direction of the magnetic field as the z-direction, $\vec{B} = B\hat{z}$. The effective potential can be generalized to this case by a simple redefinition of the dispersion relations of the fields in the presence of $\vec{B}$, using the minimal coupling shift in

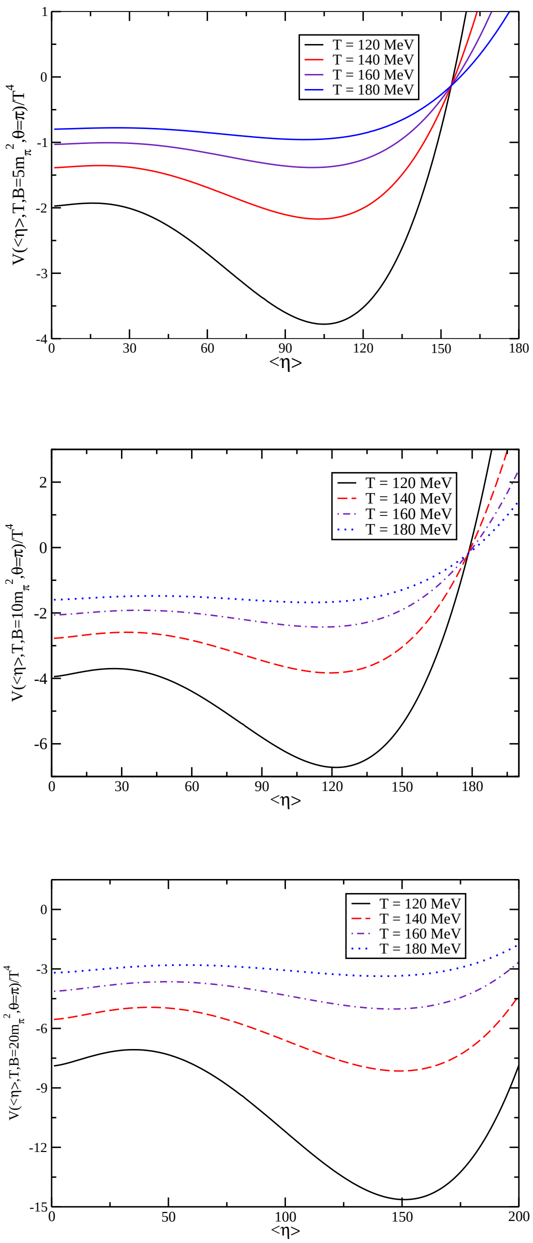


FIG. 9: Effective potential normalized by the temperature for $\theta = \pi$ in the $\langle \eta \rangle$ (in MeV) direction. The plots, from the top to the bottom of the figure, correspond to $B = 5m_\pi^2$, $B = 10m_\pi^2$, and $B = 20m_\pi^2$.

the gradient and the field equations of motion. For this purpose, it is convenient to choose the gauge such that $A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)$. Decomposing the fields into their Fourier modes, one arrives at eigenvalue equations which have the same form as the Schrödinger equation for a harmonic oscillator potential, whose eigenmodes correspond to the well-known Landau levels. The latter provide the new dispersion relations

$$p_{0n}^2 = p_z^2 + m^2 + (2n+1)|q|B\,; \tag{10}$$
$$p_{0n}^2 = p_z^2 + m^2 + (2n+1-\sigma)|q|B\,. \tag{11}$$

for scalars and fermions, respectively, n being an integer, q the electric charge, and σ the sign of the spin. It is also straightforward to show that integrals over four momenta and thermal sum-integrals acquire the following forms, respectively:

$$\int \frac{d^4k}{(2\pi)^4} \mapsto \frac{|q|B}{2\pi} \sum_{n=0}^{\infty} \int \frac{dk_0}{2\pi} \frac{dk_z}{2\pi}\,, \tag{12}$$
$$T\sum_\ell \int \frac{d^3k}{(2\pi)^3} \mapsto \frac{|q|BT}{2\pi} \sum_\ell \sum_{n=0}^{\infty} \int \frac{dk_z}{2\pi}\,, \tag{13}$$

where n represents the different Landau levels and ℓ stands for the Matsubara frequency indices [49].

In our effective model, the vacuum piece of the potential will be modified by the magnetic field through the coupling of the field to charged pions. To one loop, and in the limit of high B, $eB \gg m_\pi^2$, one obtains (ignoring contributions independent of the condensates) [29]

$$V_{\pi^+}^V + V_{\pi^-}^V = -\frac{2m_\pi^2 eB}{32\pi^2} \log 2\,. \tag{14}$$

Thermal corrections are provided by pions and quarks. However, the pion thermal contribution as well as part of the quark thermal contribution are exponentially suppressed for high magnetic fields, as has been shown in Ref. [29]. The only part of the quark thermal part that contributes is

$$V_q^T = -N_c \frac{eBT^2}{2\pi^2} \left[\int_{-\infty}^{+\infty} dx \ \ln\left(1 + e^{-\sqrt{x^2 + M_q^2/T^2}}\right) \right]\,, \tag{15}$$

where $N_c = 3$ is the number of colors. Therefore, the effective potential computed in the previous section is corrected by the contributions in (14) and (15) in the presence of a strong homogeneous magnetic background.

For $\theta = 0$, i.e. in the case of the CP-even linear sigma model, the situation is the same as the one we investigated in Ref. [29]. Initially, there are two minima in the $\langle \sigma \rangle$ direction, one global and one local. As the temperature increases, they move towards the center, approximately restoring chiral symmetry (not fully restored since there is an explicit breaking term in the effective potential, due to the nonzero quark masses). For large enough values of the magnetic background, the nature of the phase transition is altered, becoming of first order instead of a crossover, in accordance with the findings of Ref. [29]. In Fig. 7 we show contour plots of the full effective potential at $\theta = 0$ for different values of the magnetic field. In each plot we display the potential close to the critical temperature, so that the case of $B = 10m_\pi^2$ is plotted at $T = 120$ MeV, while the case of $B = 20m_\pi^2$ is plotted at $T = 100$ MeV. For $B = 5m_\pi^2$, the

barrier is quite small and the critical temperature higher, as discussed in Ref. [29].

Augmenting the value of θ to π, we can study the behavior of the η condensate. As in the case in the absence of a magnetic background, this condensate undergoes a first-order phase transition. Moreover, in the presence of a strong magnetic field, its critical temperature is strongly affected. Analogously to our findings in Ref. [29] for the CP-even case, a field $B = 5m_\pi^2$ increases the critical temperature, but now for the η condensate. The behavior of the critical temperature for stronger values of the field is analogous to the one of $\langle\sigma\rangle$, i.e. it drops considerably. In Fig. 8 we plot the effective potential at temperature $T = 195$ MeV. For $B = 5m_\pi^2$, this temperature is not enough to take the system to the critical region. On the other hand, for $B = 10m_\pi^2$ this temperature is close to the critical temperature, and for $B = 20m_\pi^2$ it is larger than the critical value. This can also be clearly seen in Fig. 9, where we display slices of the effective potential in the η direction for different values of magnetic field and temperature.

Besides modifying the value of T_c, the presence of a strong magnetic background deepens the absolute minimum and favors a first-order transition.

V. CONCLUSIONS AND OUTLOOK

The possibility of probing the vacuum structure of QCD in high-energy heavy ion collisions, bringing some light to the understanding of the strong CP problem, is a very exciting prospect. The consideration of the theoretical description of the environment that might be produced under the appropriate experimental conditions, especially the major role played by the presence of a strong magnetic background, has already brought up very interesting phenomena, such as the chiral magnetic effect [10] and the possibility of converting the nature of the chiral transition from a crossover to a first-order phase transition [29].

The complete physical scenario in heavy ion collisions is rather complicated [8, 9, 10], so that it is more prudent to consider, theoretically, the role of each relevant ingredient separately, aiming at a more complete picture at the end. Previously, we have investigated the effect of a strong magnetic background on the nature of the chiral transition [29], obtaining remarkable effects as mentioned above, and opening a new line of possibilities in the study of the phase structure of strong interactions. In this paper, we investigated how the chiral transition, more specifically how the effective potential written in terms of the condensates $\langle\sigma\rangle$ and $\langle\eta\rangle$, is altered when one includes the possibility of strong CP violation and a strong magnetic background. For this purpose we built a CP-odd extension of the linear sigma model coupled with two flavors of quarks at finite temperature and zero density. The inclusion of magnetic effects is implemented via a redefinition of the dispersion relations.

We found that, in the absence of magnetic fields, the σ condensate behaves pretty much like it does in the usual CP-even linear sigma model, changing from a phase where chiral symmetry is broken ($\langle\sigma\rangle \neq 0$) to a phase where it is approximately restored ($\langle\sigma\rangle \approx 0$) via a crossover as the temperature is increased. For non-vanishing values of the parameter θ, a condensate of the field η builds up for low temperatures, breaking CP. This condensate is trapped by a barrier in the effective potential, indicating the presence of metastable states that violate CP, which is a relevant point for the possibility of observing CP-odd domains in high-energy heavy ion collisions. We should remark that these metastable states were not found in a detailed study of the phase diagram defined in terms of the strength of the 't Hooft determinant, quark masses, temperature and chemical potential that has been performed for $\theta = \pi$ within the two-flavor NJL model [28]. The reason for this discrepancy has its grounds in the difference in physical content of the two effective theories, and is currently under investigation [50].

The main effect of a high magnetic background on the chiral transition is turning it into a first-order transition, even in the case of the CP-odd linear sigma model, deepening the absolute minimum of the effective potential. The behavior of the critical temperature is also affected in a non-trivial way, first going up and then dropping for larger values of the magnetic field, as was also observed in Ref. [29].

In order to address the physical scenario in the case of noncentral heavy ion collisions at RHIC and the LHC, one has to incorporate the effects from nonzero chiral and baryonic chemical potentials [10]. It is also crucial to describe the nonequilibrium dynamics of the formation of the condensates, which will provide the relevant time scales and give information on the actual possibility of measuring effects coming from the formation of CP-odd domains. This issues will be considered in a future publication [51].

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